4.1 Inverse Fns.

$$\frac{1}{f(x)} = \left[f(x)\right]^{-1}$$

$$\cancel{f^{-1}(x) = \frac{1}{f(x)}}$$

$$f(x)^{-2}$$

$$\frac{1}{\left(f(x)\right)^2}$$

Let $g(x) = f^{-1}(x)$ then what is $g^{-1}(x)$

To solve for an inverse $f^{\underline{n}}$..

$$y = x^2 + 2$$

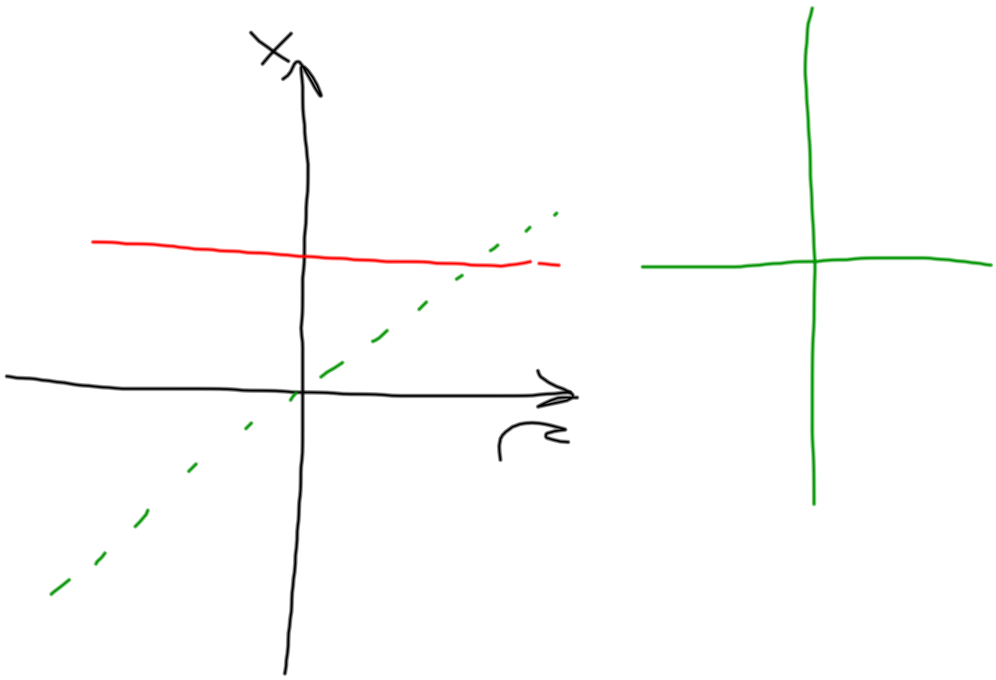
$$x = y^2 + 2$$

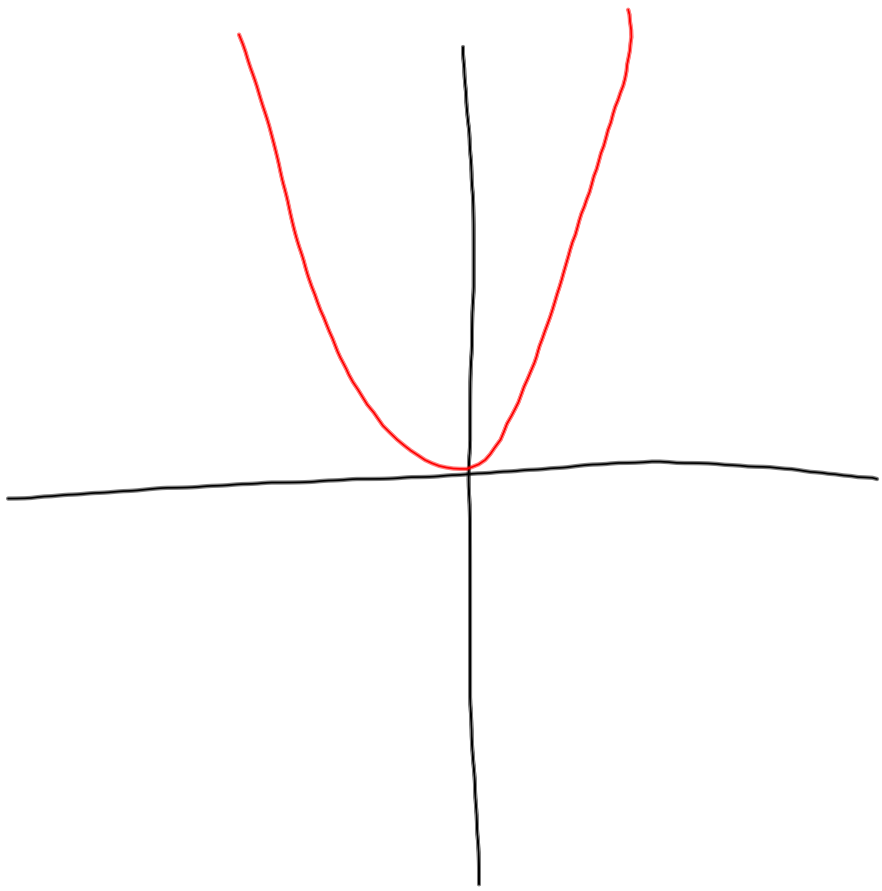
$$x - 2 = y^2$$

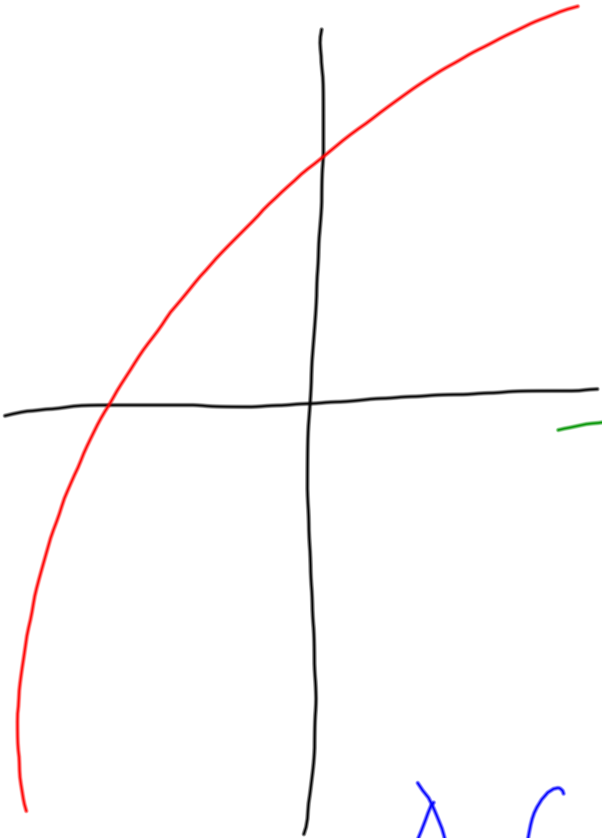
$$y = \sqrt{x-2}$$
$$y = -\sqrt{x-2}$$

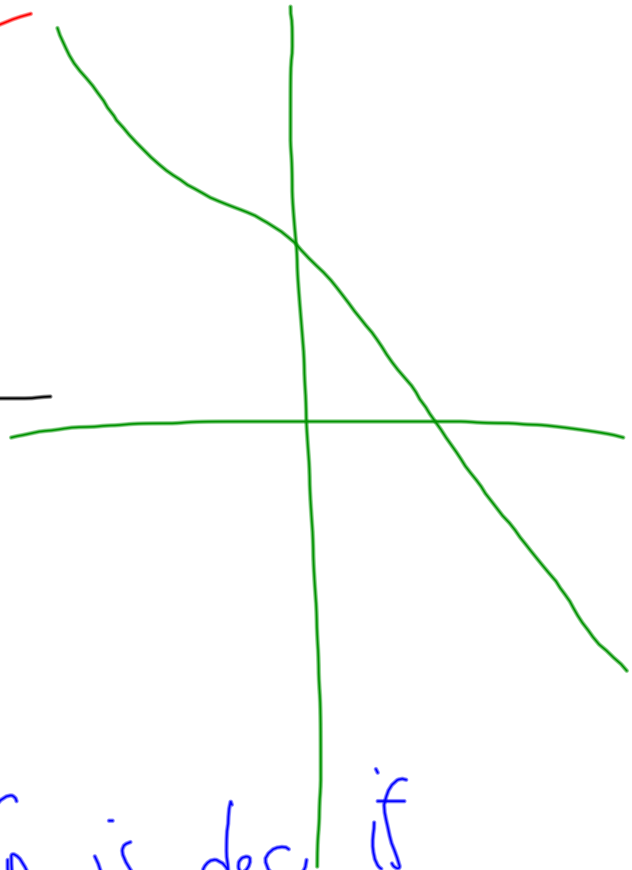
Inverse f^{ns} vs. Inverse Relations

How do we tell if a f^n's Inverse is also a f^n?









A fn is dec if

$f(x) \leq f(y) \ \forall \ x > y$

A fn is STRICTLY dec if

$f(x) < f(y) \ \forall \ x > y$

"for every"

$f^{-1}(x)$ is the inverse f^n of $f(x)$ iff

1) $f^{-1}(f(x)) = x$ for every x in domain of $f(x)$

2) $f(f^{-1}(x)) = x$ for every x in domain of $f^{-1}(x)$

What is the derivative of $f^{-1}(x)$ related to?

$y = x^2$

when $x = 2.5$

$y = (2.5)^2 = 6.25$

$m_{tan} = 5$

$y = \sqrt{x}$

when $x = 6.25$

$y = 2.5$

$m_{tan} = \frac{1}{5} = .2$

$$f\left(f^{-1}(x)\right) = x$$

$\mathrm{I}_{mp}\,\mathrm{D}_{if}$

$$f'\left(f^{-1}(x)\right) \cdot \left[f^{-1}(x)\right]' = 1$$

$$\left[f^{-1}(x)\right]' = \frac{1}{f'\left(f^{-1}(x)\right)}$$

1) $f(x) = x^4$; $g(x) = \sqrt[4]{x}$

Are they inverses?

[Use the composition def^n]

$f(g(x)) = f(f^{-1}(x)) = (\sqrt[4]{x})^4 = x$ ✓

Domain of $\sqrt[4]{x}$ is $[0, \infty)$

$g(f(x)) = \sqrt[4]{x^4} = x$ ✗

Domain of x^4 is all reals

$\sqrt[4]{(-1)^4} = +1$

$(-1)^4 = +1$

$\sqrt[4]{1} = +1$

are $f(x)=x^3$ and $g(x)=\sqrt[3]{x}$ inverses (use composition def^n)

$f(g(x)) = (\sqrt[3]{x})^3 = x$ ✓

Domain of $g(x)$ is: all reals

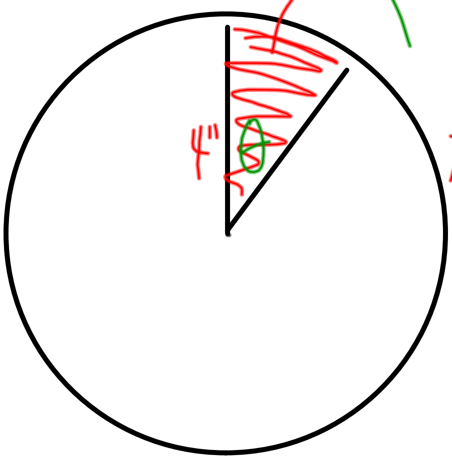
$g(f(x)) = \sqrt[3]{(x^3)} = x$ ✓

Domain of $f(x)$ is: all reals

3.7
11

$$\frac{\text{Area}_{\text{sector}}}{\text{Area}_{\text{circle}}} = \frac{\text{Angle}_{\text{sector}}}{\text{Angle}_{\text{whole circle}}}$$

Area



$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A = \frac{\theta}{2\pi}(\pi r^2)$$

$$A = 8\theta$$

$$\frac{dA}{dt} = 8\frac{d\theta}{dt} = 8\,\text{in}^2\left(\frac{2\pi \text{ radians}}{\text{hr}}\right) = 16\pi \frac{\text{in}^2}{\text{hr}}$$

$$16\pi \frac{\text{in}^2}{\text{hr}} \cdot \frac{\text{hr}}{60\,\text{min}}$$

$$\frac{16\pi}{60}\frac{\text{in}^2}{\text{min}} = \frac{4\pi}{15}\frac{\text{in}^2}{\text{min}}$$

3.7/4)

$$\dot{x}^2 + \dot{y}^2 = 2x$$

$$\frac{dx}{dt} = -2 \quad (1,1)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2$$

Not the correct derivative!

Forgot dx/dt

$$2(1)(-2) + 2(1)\left(\frac{dy}{dt}\right) = 2$$

$$-4 + 2\frac{dy}{dt} = 2$$

$$+4 \qquad +4$$

$$\frac{2\frac{dy}{dt}}{2} = \frac{6}{2} = 3$$

4.1 Inverse Fns

switch y & x.

$y = f(x)$ . . . .

interchange y & x & "solve"

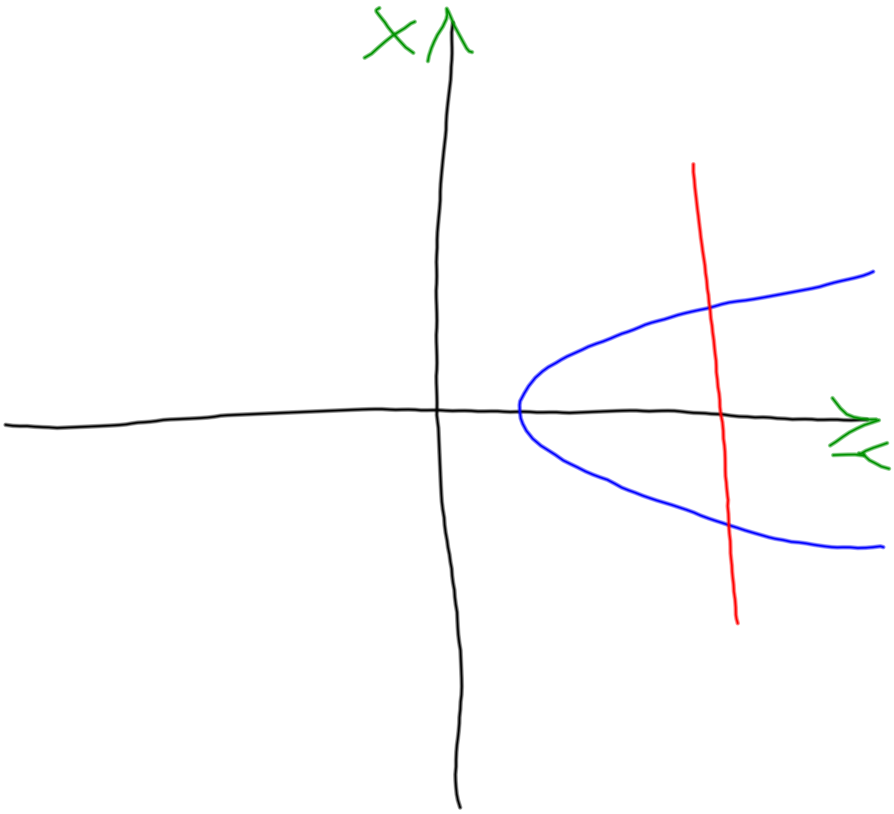
$x = f(y)$ . . .

$y = x^2 + 2$

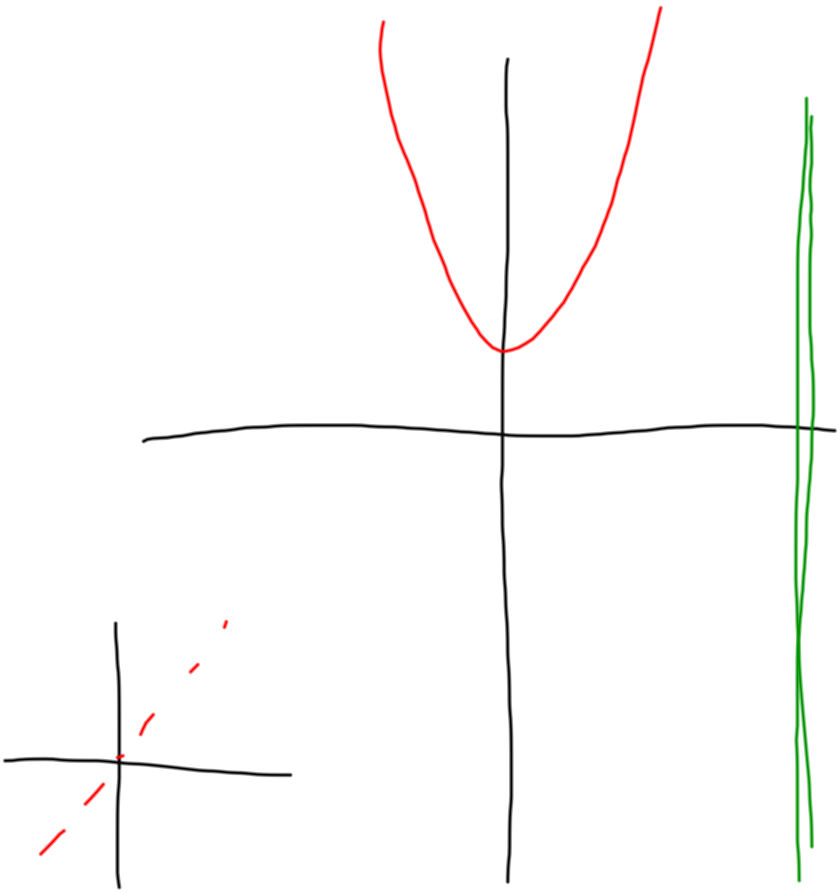
$x = y^2 + 2$

$x - 2 = y^2$

$y = \pm\sqrt{x-2}$ (+ or −)



How do I tell if the inverse of a f^n will be a f^n also, or just a RELATION



A fn whose inverse is also a fn is called

One-to-One

or

invertible

or

1-1

f and g are inverse f^{ns}. iff

1) $f(g(x)) = x$ [for every x in domain of g] i.e. 2 way fact

2) $g(f(x)) = x$ [for every x in domain of f]

Determine whether $f(x) = x^4$ and $g(x) = \sqrt[4]{x}$ are inverses

$f(g(x))$

$(\sqrt[4]{x})^4 = x$

for every x in domain of $\sqrt[4]{x}$

Domain of $\sqrt[4]{x}$: $[0, \infty)$

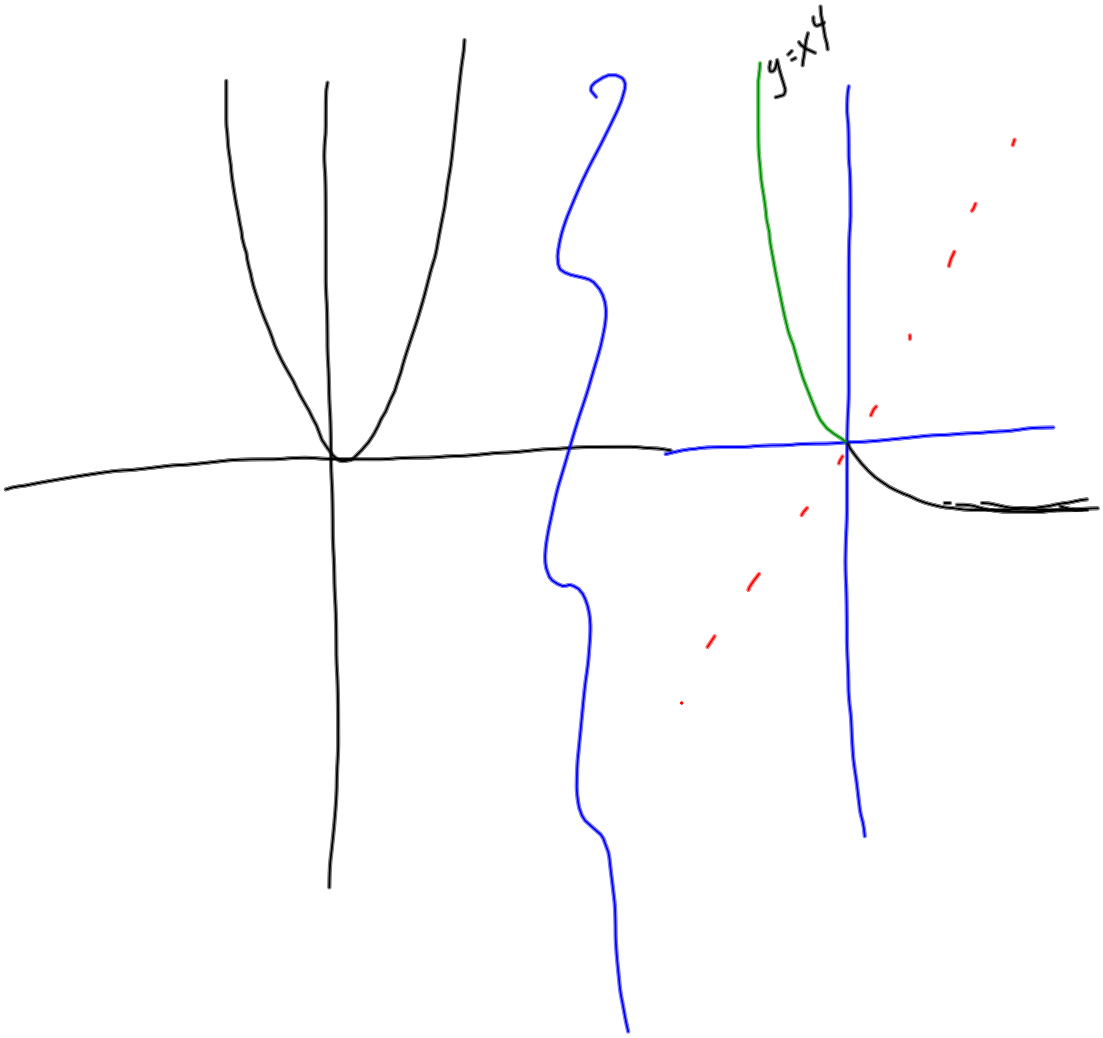
$g(f(x))$

$\sqrt[4]{x^4} = x$ ~~NEIN~~

for every x in domain of x^4

Domain of x^4 : all real

Range of x^4 : $[0, \infty)$


y=x4

Notation for f inverse...

$f^{-1}(x)$ denotes INVERSE

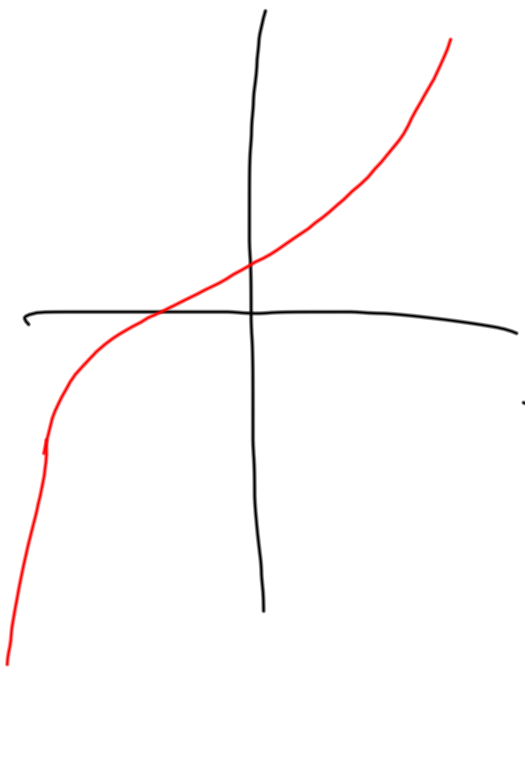
it is NEVER $\frac{1}{f(x)}$

Also note...

$f^{-1}(x)$ and $\frac{1}{f(x)}$ are hardly ever the same

example to follow

There are certain f^{ns} that always have inverse f^{ns}.



A f^n f is increasing if

$f(x) \geq f(y)$ whenever $x > y$

A f^n f is STRICTLY increasing if

$f(x) > f(y)$ whenever $x > y$

Is the derivative of an inverse related to the derivative of the original?

$y = x^2$

so $x = 2.5$

gives $y = (2.5)^2 = 6.25$

$\left(2\frac{1}{2}, 6\frac{1}{4}\right)$

$m = 5$

$y = \sqrt{x}$

So $x = 6.25$

gives $y = \sqrt{6.25} = 2.5$

$\left(6\frac{1}{4}, 2\frac{1}{2}\right)$

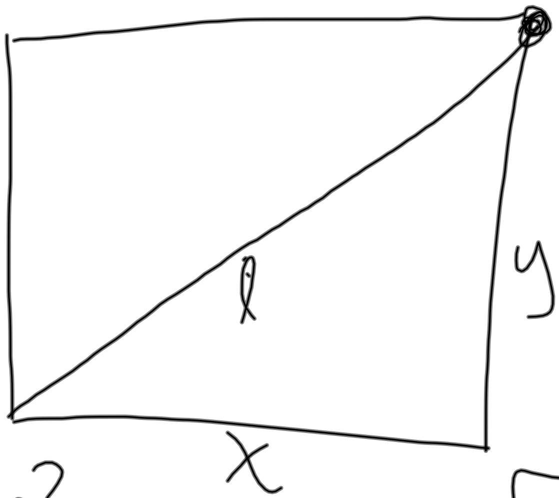
$m = .2 = \frac{1}{5}$

$$f\left(f^{-1}(x)\right) = x$$

$$f'\left(f^{-1}(x)\right) \cdot \left[f^{-1}(x)\right]' = 1$$

$$\left[f^{-1}(x)\right]' = \frac{1}{f'\left(f^{-1}(x)\right)}$$

3.7/8)



$x^2+y^2=l^2$ $\qquad$ $l=\sqrt{x^2+y^2}$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2l\frac{dl}{dt}$